International Statistical Review (2004), **72,** 3, 409–422, Printed in Wales by Cambrian Printers

Statistical Inference under Symmetry

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Summary

We explore the consequences of adjoining a symmetry group to a statistical model. Group actions are first induced on the sample space, and then on the parameter space. It is argued that the right invariant measure induced by the group on the parameter space is a natural non-informative prior for the parameters of the model. The permissible sub-parameters are introduced, i.e., the subparameters upon which group actions can be defined. Equivariant estimators are similarly defined. Orbits of the group are defined on the sample space and on the parameter space; in particular the group action is called transitive when there is only one orbit. Credibility sets and confidence sets are shown (under right invariant prior and assuming transitivity on the parameter space) to be equal when defined by permissible sub-parameters and constructed from equivariant estimators. The effect of different choices of transformation group is illustrated by examples, and properties of the orbits on the sample space and on the parameter space are discussed. It is argued that model reduction should be constrained to one or several orbits of the group. Using this and other natural criteria and concepts, among them concepts related to design of experiments under symmetry, leads to links towards chemometrical prediction methods and towards the foundation of quantum theory.

Key words and phrases: Confidence sets; Credibility sets; Group; Invariance; Invariant measure; Loss; Non-informative prior; Optimal estimator; Objective Bayes; Orbit; Partial least squares regression; Permissible sub-parameter; Pitman estimator; Quantum mechanics; Right invariant prior; Risk; Symmetry; Transitivity.