

## Wavelet analysis of conservative cascades

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A conservative cascade is an iterative process that fragments a given set into smaller and smaller pieces according to a rule which preserves the total mass of the initial set at each stage of the construction almost surely and not just in expectation. Motivated by the importance of conservative cascades in analysing multifractal behaviour of measured Internet traffic traces, we consider wavelet-based statistical techniques for inference about the cascade generator, the random mechanism determining the redistribution of the set's mass at each iteration. We provide two estimators of the structure function, one asymptotically biased and one not, and prove consistency and asymptotic normality in a range of values of the argument of the structure function less than a critical value. Simulation experiments illustrate the asymptotic properties of these estimators for values of the argument both below and above the critical value. Beyond the critical value, the estimators are shown not to be asymptotically consistent.

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