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Minimax or maxisets?

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We discuss a new way of evaluating the performance of a statistical estimation procedure. This consists of investigating the maximal set where a given procedure has a given rate of convergence. Although the setting is not vastly different from the minimax context, it is in a sense less pessimistic and provides a functional set which is authentically connected to the procedure and the model. We also investigate more traditional concerns about procedures: oracle inequalities. Difficulties arise in the practical definition of this notion when the loss function is not the L_2 norm. We explain these difficulties and suggest a new definition in the cases of L_p norms and pointwise estimation. We investigate the connections between maxisets and local oracle inequalities, and prove that verifying a local oracle inequality implies that the maxiset automatically contains a prescribed set linked with the oracle inequality. We have investigated the consequences of this statement on well-known efficient adaptive methods: wavelet thresholding and local bandwidth selection. We prove local oracle inequalities for these methods and draw conclusions about the maxisets associated with them.

Keywords: adaptive methods; oracle inequalities; saturation sets