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Nonlinear estimation over weak Besov spaces and minimax Bayes method

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Weak Besov spaces play an important role in statistics as maxisets of classical procedures or for measuring the sparsity of signals. The goal of this paper is to study weak Besov balls $\mathcal{WB}_{s,p,q}(C)$ from the statistical point of view by using the minimax Bayes method. In particular, we compare weak and strong Besov balls statistically. By building an optimal Bayes wavelet thresholding rule, we first establish that, under suitable conditions, the rate of convergence of the minimax risk for $\mathcal{WB}_{s,p,q}(C)$ is the same as for the strong Besov ball $\mathcal{B}_{s,p,q}(C)$ that is contained in $\mathcal{WB}_{s,p,q}(C)$. However, we show that the asymptotically least favourable priors of $\mathcal{WB}_{s,p,q}(C)$ that are based on Pareto distributions cannot be asymptotically least favourable priors for $\mathcal{B}_{s,p,q}(C)$. Finally, we present sample paths of such priors that provide representations of the worst functions to be estimated for classical procedures and we give an interpretation of the roles of the parameters s, p and q of $\mathcal{WB}_{s,p,q}(C)$.

Keywords: asymptotically least favourable priors; Bayes method; minimax risk; rate of convergence; thresholding rules; weak Besov spaces