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Estimation of the extreme-value index and generalized quantile plots

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In extreme-value analysis, a central topic is the adaptive estimation of the extreme-value index γ. Hitherto, most of the attention in this area has been devoted to the case $\gamma > 0$, that is, when $\bar{F}$ is a regularly varying function with index $-1/\gamma$. In addition to the well-known Hill estimator, many other estimators are currently available. Among the most important are the kernel-type estimators and the weighted least-squares slope estimators based on the Pareto quantile plot or the Zipf plot, as reviewed by Csörgő and Viharos. Using an exponential regression model (ERM) for spacings between successive extreme order statistics, both Beirlant *et al.* and Feuerverger and Hall introduced bias-reduced estimators.

For the case where γ is real, Hill's estimator has been generalized to a moment-type estimator by Dekkers *et al.* Alternatively, Beirlant *et al.* introduced a Hill-type estimator that is based on the generalized quantile plot. Another popular estimation method follows from maximum likelihood estimation applied to the generalizations of the Pareto distribution. In the present paper, slope estimators for $\gamma > 0$ are generalized to the case where γ is real-valued. This is accomplished by replacing the Zipf plot by a generalized quantile plot. We make an asymptotic comparison of our estimator with the moment estimator and with the maximum likelihood estimator. A case study illustrates our findings. Finally, we offer a regression model that generalizes the ERM in that it allows the construction of bias-reduced estimators. Moreover, the model provides an adaptive selection rule for the number of extremes needed in several of the existing estimators.

Keywords: bias; extreme-value index; least squares; mean squared error; quantile plots