

Strong consistency of the maximum likelihood estimator for finite mixtures of location–scale distributions when the scale parameters are exponentially small

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In a finite mixture of location–scale distributions the maximum likelihood estimator does not exist because of the unboundedness of the likelihood function when the scale parameter of some mixture component approaches zero. In order to study the strong consistency of the maximum likelihood estimator, we consider the case where the scale parameters of the component distributions are restricted from below by c_n , where $\{c_n\}$ is a sequence of positive real numbers which tend to zero as the sample size n increases. We prove that under mild regularity conditions the maximum likelihood estimator is strongly consistent if the scale parameters are restricted from below by $c_n = \exp(-n^d)$, $0 < d < 1$.

Keywords: consistency; maximum likelihood estimator; mixture distribution