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A characterization of Poisson–Gaussian families by generalized variance

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We show that if the generalized variance of an infinitely divisible natural exponential family $F = F(\mu)$ in a d-dimensional linear space is of the form $\det K''_\mu(\boldsymbol{\theta}) = \exp(\boldsymbol{\theta}^{\mathrm{T}}\mathbf{b} + c)$, then there exists k in $\{0, 1, \ldots, d\}$ such that F is a product of k univariate Poisson and $(d-k)$-variate Gaussian families. In proving this fact, we use a suitable representation of the generalized variance as a Laplace transform and the result, due to Jörgens, Calabi and Pogorelov, that any strictly convex smooth function f defined on the whole of $\mathbb{R}^d$ such that $\det f''(\boldsymbol{\theta})$ is a positive constant must be a quadratic form.

Keywords: affine variance function; determinant; infinitely divisible measure; Laplace transform; Monge–Ampère equation; r-reducibility