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Efficient estimation of stochastic volatility using noisy observations: a multi-scale approach

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With the availability of high-frequency financial data, nonparametric estimation of the volatility of an asset return process becomes feasible. A major problem is how to estimate the volatility consistently and efficiently, when the observed asset returns contain error or noise, for example, in the form of microstructure noise. The issue of consistency has been addressed in the recent literature. However, the resulting estimator is not efficient. In work by Zhang, Myland and Aït-Sahalia, the best estimator converges to the true volatility only at the rate of $n^{-1/6}$. In this paper, we propose an estimator, the multi-scale realized volatility (MSRV), which converges to the true volatility at the rate of $n^{-1/4}$, which is the best attainable. We show a central limit theorem for the MSRV estimator, which permits intervals to be set for the true integrated volatility on the basis of the MSRV.

Keywords: consistency; dependent noise; discrete observation; efficiency; Itô process; microstructure noise; observation error; rate of convergence; realized volatility