Bernoulli **12**(4), 2006, 571–581 ISSN 1350-7265 © 2006 ISI/BS

Quasi-stationary distributions for Lévy processes

ANDREAS E. KYPRIANOU[1] and ZBIGNIEW PALMOWSKI[2,3]

[1] *School of Mathematical and Computer Sciences, Heriot-Watt University, Edinburgh EH14 4AS, UK. E-mail: kyprianou@ma.hw.ac.uk*
[2] *Mathematical Institute, University of Wrocław, pl. Grunwaldzki 2/4, 50-384 Wrocław, Poland. E-mail: zpalma@math.uni.wroc.pl*
[3] *Mathematical Institute, Utrecht University, P.O. Box 80 010, 3508 TA Utrecht, Netherlands. E-mail: palmowski@math.uu.nl*

In recent years there has been some focus in work by Bertoin, Chaumont and Doney on the behaviour of one-dimensional Lévy processes and random walks conditioned to stay positive. The resulting conditioned process is transient. In earlier literature, however, one encounters for special classes of random walks and Lévy processes a similar, but nonetheless different, type of asymptotic conditioning to stay positive which results in a limiting quasi-stationary distribution. We extend this theme into the general setting of a Lévy process fulfilling certain types of conditions which are analogues of known classes in the random walk literature. Our results generalize those of E.K. Kyprianou for special types of one-sided compound Poisson processes with drift and of Martínez and San Martín for Brownian motion with drift, and complement the results due to Iglehart, Doney, and Bertoin and Doney for random walks.

Keywords: conditioning; fluctuation theory; Lévy processes; quasi-stationary distribution