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# A complete characterization of local martingales which are functions of Brownian motion and its maximum

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We prove the max-martingale conjecture of Obłój and Yor. We show that for a continuous local martingale $(N_t : t \geq 0)$ and a function $H : \mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}$, $H(N_t, \sup_{s \leq t} N_s)$ is a local martingale if and only if there exists a locally integrable function $f$ such that $H(x, y) = \int_0^y f(s)\mathrm{d}s - f(y)(x - y) + H(0, 0)$. This readily implies, via Lévy's equivalence theorem, an analogous result with the maximum process replaced by the local time at 0.

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