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Pooling strategies for St Petersburg gamblers

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Peter offers to play exactly one St Petersburg game with each of $n \geq 2$ players, $\text{Paul}_1, \ldots, \text{Paul}_n$, whose conceivable pooling strategies are described by all possible probability distributions $\boldsymbol{p}_n = (p_{1,n}, \ldots, p_{n,n})$. Comparing infinite expectations, we characterize among all $\boldsymbol{p}_n$ those admissible strategies for which the pooled winnings, each distributed as $V_{\boldsymbol{p}_n} = \sum_{k=1}^{n} p_{k,n} X_k$, yield a finite added value for each and every one of $\text{Paul}_1, \ldots, \text{Paul}_n$ in comparison with their individual winnings $X_1, \ldots, X_n$, even though their total winnings $S_n = X_1 + \ldots + X_n$ is the same. We show that the added value of an admissible $\boldsymbol{p}_n$ is just its entropy $H(\boldsymbol{p}_n)$, and we determine the best admissible strategy $\boldsymbol{p}_n^*$. Moreover, for every $n \geq 2$ and $\boldsymbol{p}_n$ we construct semistable approximations to $S_{\boldsymbol{p}_n} = V_{\boldsymbol{p}_n} - H(\boldsymbol{p}_n)$. We show in particular that $S_{\boldsymbol{p}_n}$ has a proper semistable asymptotic distribution as $n \to \infty$ along the entire sequence of natural numbers whenever $\max\{p_{1,n}, \ldots, p_{n,n}\} \to 0$ for a sequence $\boldsymbol{p}_n$ of admissible strategies, which is in sharp contrast to S_n/n, and the rate of convergence is very fast for $S_{\boldsymbol{p}_n^*}$.

Keywords: added value; asymptotic distributions; best admissible pooling strategies; comparison of infinite expectations; several players; St Petersburg games