Compound Supersaturated Designs

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1. INTRODUCTION

In most factorial experiments the number of effects (assuming interactions are absent) do not exceed the number of observations/runs, which guarantees the estimation of all the main-effects simultaneously. There are some practical situations that include large number of potentially relevant factors of which only a few are expected to have clear effects on the response of interest. However, we do not know which factors are active. The basic problem here is to identify these few active factors in an economic way. If we assume no interaction between the factors and if the number of active factors is assumed to be small, a supersaturated design can solve the above problem. Studies of supersaturated designs have received considerable interests in recent years; see for example, Tang and Wu (1997), Wu (1993), Yamada and Lin (1997).

Suppose in an experimental situation we have a large number of factors to examine and the factors are reasonably clustered into several correlated groups. However, it is not unusual to have one of the following situations- (i) only a few factors in each group are found to be active, and (ii) only a few groups of factors are found to be active.

Our primary objective is then to identify either the active factors in each group or the active groups of factors. Finally these factors are then studied further. Watson (1961) considered the problem stated in (ii) and suggested an alternative idea by grouping the factors into a smaller number of groups and treating each group of factors as a single factor, called grouped factor. In the course of the analysis, if a grouped factor is found to have a significant effect, then the factors in the group will be studied in a subsequent experiment. The present paper deals with problem (i).

2. MAIN RESULT

Let there be M 2-level factors that are grouped into m groups. The *ith* group consists of r_i factors. Let **Y** be the observational vector. Then the linear model considered here is similar to that of Chatterjee and Mukerjee (1986), i.e.,

E(**Y**) = X❶ , Disp(Y) = ⑨^{2}I,

where ❶ = (❶$_0$,❶ '$_1$,❶ '$_2$,. .,❶ '$_m$)', ❶$_0$ is the general mean and ❶$_I$ consists of the main effects of the factors belong- ing to the *ith* group of factors. Suppose ❶$_0$ is not known and also suppose it is known that for each i (I = 1,. .,m), at most k_i elements of ❶$_I$ are active where k_i is small compared to $r_{i..}$ Our objective is to provide a supersaturated design that will identify the active factors belonging to each group and also to estimate them along with ⓿$_0$.

Let S_i be a subset of n_i level combinations of the factors belonging to the ith group that gives a supersaturated design that allows the detection and estimation of the possibly present at best k_i active factors along with the general mean. Let, for I=1,2,…,m, w_i be any singleton subset of S_i and $W_i = S_i – w_i$. Define

$$S = (w_1 \& w_2 \& \ldots \& w_m) U(U_i(w_i \& \ldots \& w_{(i-1)} \& W_i \& w_{(i+1)} \& \ldots \& w_m)),$$

where & denotes the symbolic direct product.

Theorem 2.1. The set of level combinations of the factors given by S gives a supersaturated design for detecting and estimating the possibly present active factors among all the groups of factors along with the general mean in the noiseless case.

Proof. This theorem can be proved along the line of Chatterjee and Mukerjee (1986) through a great deal of mathematics.

Remark 2.1. The existence of supersaturated designs S_1, S_2, …, S_m implies the existence of a supersaturated design S described above.

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RESUME

Celte page considere le probleme de construire les dessins supersaturé qui permaltent la detetion et l’ evaluation des facteurs à les groupes differents de facteurs potentiellement utiles. Une methode de telle construction est proposèe ici.