Multivariate Log – Normal Distribution

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1-Introduction

Multivariate distributions have important role in economics and statistics. Since variables in economics, psychology, and reliability are positive; multivariate positive distributions should be considered. Johnson, and Kotz (1972) cited some multivariate positive distributions such as multivariate Gamma, Beta, Pareto, F distribution which have very complicated forms and are not easy to use in Economical and Reliability investigations.

Jeevavand (1997) drive the Bayesian estimate of $P(X_1 < X_2)$ in bivariate Pareto distribution. Tong (1980) developed the area of probability inequalities in multivariate distributions. Tarmast (1997) used multivariate normal distribution to obtain bounds for the multiple integral P (B<X<C).

Here, multivariate log - normal distribution is defined and its mean and covariance matrix are obtained and their estimates are calculated. The application of the multivariate log - normal distribution in reliability is mentioned.

2-Multivariate Log-Normal Distribution

Let $X= [X_1, X_2, \ldots, X_p]$ be a p-component random vector having multivariate Normal distribution with mean ν and covariance matrix $D = (d_{ij})$. Now we use the transformation $Y_i = \exp(X_i)$ and define $Y=[Y_1, Y_2, \ldots, Y_p]$. The density of Y is multivariate log - normal distribution and has the following form:

$$f_Y(y)=(2\Pi)^{-P/2} * |D|^{-1/2} * [y_1, y_2, \ldots, y_p]^{-1} * \mathrm{Exp}[-(\mathrm{Lny} - \nu)' D^{-1} (\mathrm{Lny} - \nu)/2] \quad 0<y_i<\infty \qquad (1).$$

Where $\mathrm{Lny} = [\mathrm{Lny}_1, \mathrm{Lny}_2, \ldots, \mathrm{Lny}_p]$ is a p-component column vector and $y_i = \exp(x_i)$.

3-Mean and Covariance Matrix

Let Y be a p-component vector and have multivariate log - normal distribution. Mean of Y is :

$$\mu=E(Y)=[\mu_1, \mu_2, \ldots \mu_P] \qquad \mu_i=\exp(\nu_i+0.5d_{ii})$$

(2)

Where d_{ii} is ith. Diagonal element of matrix D. Covariance matrix of Y is :

$$\Sigma=E[(Y-\mu)(Y-\mu)']=(\sigma_{ij}) \qquad \sigma_{ij}=[\exp[(\nu_I+\nu_j)+(d_{ii}+d_{jj})/2]]*[\exp(d_{ij})-1] \qquad (3).$$

Where d_{ij} is the ijth. Element of D.It is clear that if X_1, X2, …. Xp are independent, then Y_1, Y_2 , …. Yp are also independent and vice versa.

4-Estimation

Let $Y^{(1)}, Y^{(2)}, \ldots, Y^{(n)}$ be a random sample of p-component vectors having multivariate log-normal distribution and $X^{(1)}, X^{(2)}, \ldots, X^{(n)}$ be the corresponding random sample of vectors having multivariate normal distribution respectively.The maximum likelihood estimators of μ and Σ are considered and obtained. To find these estimators the maximum likelihood estimators of ν_i, d_{ii}, d_{ij} should be found. Suppose $\underline{X}_i$, S_{ii}, S_{ij} are the maximum likelihood estimators of ν_i, d_{ii}, d_{ij} respectively.

$\overline{X}_i = (\sum X_{ij})/n$ $\quad S_{ii}=\sum(X_{ij} - \overline{X}i)^2$ $\quad S_{ij}=\sum (X_{ik} - \overline{X}i)(X_{jk} - \overline{X}j) / n$

(4).

Now, the maximum likelihood estimators of μ_i, σ_{ij} can be found.

$\hat{m}_i = \exp[\bar{x}i + S_i^2/2]$ $\quad \hat{s}^2{}_i = [\exp(2\bar{x}i + S_i^2/2]*[\exp(S_i^2) - 1]$.

(5)

$\hat{s}_{ij} = [\exp[(\bar{x}_i + \bar{x}_j) + (S_i^2+S_j^2)/2)]*[\exp(S_{ij}) - 1]$

Where $\hat{m}_i$ is the ith. Element of the maximum likelihood estimator of m and $\hat{s}_{ij}$ is the ijth. element of the maximum likelihood estimator of Σ .

5-Application

Let $Y=[Y_1,Y_2, .,Y_p]$ be a p–component random vector having multivariate log- normal distribution and $B=[B_1,B_2,\ldots,B_p]$ and $C=[C_1,C_2,\ldots,C_p]$ be two given finite positive constant vectors such that $B_i \leq C_i$, then the upper and lower bound for $p(B \leq Y \leq C)$ can be found :

$$p(B \leq Y \leq C) = p(LnB \leq LnY \leq LnC) = P(B_0 \leq X \leq C_0). \quad (6).$$

Where X has multivariate normal distribution and B_0 and C_0 are given constants. The upper and lower bounds for (6) can be found .See Tarmast(1977).

6-BIBLIOGRAPHY

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7-SUMMARY

Let X be a random vector have a multivariate normal distribution. Then, Y= exp(X) is a random vector having multivariate log – normal distribution. The density distribution of Y is obtained and mean and covariance matrix of Y are estimated and the application of the distribution is mentioned.