

Estimations of Lorenz Curve and Gini Index in a Pareto Distribution

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1. Introduction

Let Y_1, Y_2, \dots, Y_n **be a simple random sample** from a Pareto distribution with the pdf:

$$f(y; \mathbf{b}) = \frac{\mathbf{b}}{\mathbf{b}-1} \cdot y^{\frac{2\mathbf{b}-1}{1-\mathbf{b}}}, \quad 1 < y, \quad 1 < \mathbf{b}, \quad (1.1)$$

which has mean \mathbf{b} and variance $\mathbf{b}(\mathbf{b}-1)^2/(2-\mathbf{b})$, if $1 < \mathbf{b} < 2$

A Pareto distribution with the pdf (1.1) is a special case of the Pareto distribution with the scale parameter $\mathbf{q} = 1$ and the shape parameter $\mathbf{k} = \mathbf{b}/(\mathbf{b}-1)$ (see Johnson(1970)), and the density

function is decreasing if $\mathbf{b} > 1$. Let $\mathbf{b} \equiv \frac{\mathbf{q}}{2\mathbf{q}-1}$ in a power-function random variable with the pdf

$f(x; \mathbf{q}) = \frac{\mathbf{q}}{1-\mathbf{q}} \cdot x^{\frac{2\mathbf{q}-1}{1-\mathbf{q}}}$, $0 < x < 1$, $1/2 < \mathbf{q} < 1$ (see Ali. etal (1999)). Then an inverse of the power-

function random variable has a Pareto distribution with the pdf (1.1). Hung & Bier(1998) considered the properties of the conjugate prior for the non-homogeneous poisson process with a power-function law intensity. Ali, Woo & Yoon(1999) studied the UMVUE for mean and right-tail probability in a power-function distribution , that is, an inverse of a Pareto random variable with the pdf (1.1).

As comparing MSE of MLE and UMVUE of the Lorenz curve and Gini index in a Pareto distribution with pdf (1.1), the UMVUE's of the Lorenz curve and Gini index tend to more efficient than their MLE.

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