Nonparametric MLE for nonstationary processes

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We consider the estimation of time varying characteristics (functions in time) of *locally* stationary processes as introduced by Dahlhaus (1997). As a special cases AR-processes with time varying coefficients are considered, and our statistical goal is to estimate the time varying parameters, or functions in time.

Our approach to this problem is based on maximizing the Whittle-likelihood over a class of function $\mathcal{F}$. For instance, when considering earthquake data it seems to be reasonable to assume that (among others) the variance is varying in time. In fact, it might be reasonable to model the variance as a function that is decreasing after the arrival of the main shock waves. It is our intention to include such types of shape restriction into the estimation procedure.

As for the analysis of the asymptotic behavior of our estimators, it turns out that there is a formal similarity to "usual" nonparametric ML-estimation where the analysis of the asymptotic behavior of the MLEs is based on modern empirical process theory (see, for instance the monograph of v.d. Geer 2000). In our case, the role of the empirical process is taken over by the (time varying) empirical spectral process.

In order to utilize this appealing similarity, we develop theory for the empirical spectral process that somehow parallels the theory for the usual empirical process. For instance, we provide a Bernstein-type exponential inequality, and use this to derive, e.g., weak convergence of the empirical spectral process to a Gaussian limit. This theory actually is of independent interest.

In special cases of estimating monotonic functions, algorithmic issues can be tackled by utilizing ideas from isotonic regression.

REFERENCES

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RESUME

Nonparametric maximum Whittle-likelihood estimation of characteristics of locally stationary time series is considered. An example is the estimation of time varying coefficients of an AR-time series. Consistency and stochastic rates of convergence for these (function) estimates are derived by utilizing the (time varying) empirical spectral process. This process comes into play, because the frequency domain is used as a vehicle to tackle the underlying problem. Theory that parallels modern empirical process theory (which is based on i.i.d.-data) is developed in order to derive the asymptotic behavior of our estimators. In special instances algorithmic issues can be tackled by utilizing ideas from isotonic regression.