Graphical Comparison of Hazards in Proportional Hazards Model

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1. Introduction and summary

The proportional hazards model (Cox, 1972) specifies the the hazard function for a subject at time t under the observed value of the covariate vector z has the following form:

$$\lambda(t|z) = \lambda_0(t)\exp(\beta' z)$$

where $\lambda_0(t)$ is a fixed baseline hazard function and β is a vector of regression parameters. Statistical inference on β is usually based on the partial likelihood function (Cox, 1975) and the conditional survival probability is estimated by Breslow(1974)'s estimator. Since this estimate concerns a single point z it is natural to consider what is the survival probability given that the covariates belong to some subset R? Is there a hazard proportionality of a subset $R_2$with respect to a subset R_1? The examples of set $R$are such that an age group, or a certain range of continuous variable. In this paper, we propose estimates of conditional cumulative hazards given some subset of covariates and graphical methods for comparing hazards in two disjoint interested subsets of covariates. Suppose that $T_i, C_i, Z_i (i = 1, ...n)$are independent and identicallty distributed and assume that T_i and C_i are independent conditional on Z_i. Let $0 < t_1 < ... < t_k$ be the distinct failure times, and $w(t_i)$be the jump size of the Kaplan-Meier estimator at t_i. Then the conditional cumulative hazard function given a subset of covariate R is consistently estimated by

$$\widehat{\Lambda}(t|Z \in R) = \frac{\sum_{t_j <= t} \widehat{P}(Z \in R|t_j) w(t_j)}{\sum_{t_i > t_j} \widehat{P}(Z \in R|t_i) w(t_i)}$$

where $\widehat{P}(Z \in R|t)$is an estimate of the conditional distribution function of Z given $T = t$ under a proportional hazards model proposed by Xu and O'Quigley(2000). Let R_1 and R_2be two disjoint interested subsets of covariates. Then the relative difference function in cumulative hazards $\widehat{\Delta}(t) = \frac{\widehat{\Lambda}(t|Z \in R_2) - \widehat{\Lambda}(t|Z \in R_1)}{\widehat{\Lambda}(t|Z \in R_1)}$ and the relative trend function in cumulative hazards $\widehat{r}(u) = \widehat{\Lambda}_2(t)(\widehat{\Lambda}_1^{-1}(t))$ are used for checking the proportionality of hazards on two interesting subsets of covariates. (See Dabrowska et al, 1989), where $\widehat{\Lambda}_1(t)$ and $\widehat{\Lambda}_2(t)$ are the estimates of conditional cumulative hazard function given $Z \in R_1$ and $Z \in R_2$, respectively. Further the asymptotic normality of this estimate is provided and a gastric cancer data is illustrated.

REFERENCES

[1] Breslow, N. (1974). Covariance analysis of censored survival data. *Biometrics*, **30**, 89-9.

[2] Cox, D.R. (1972). Regression models and life tables (with discussion). *J. R. Statist. Soc. B*, **34**, 187-202.

[3] Cox, D.R. (1975). Partial likelihood. *Biometrika*, **62**, 269-276.

[4] Dabrowska, D.M., Doksum, K.A. and Song, J.K. (1989). Graphical comparison of cumulative hazards for two populations. *Biometrika*, **76**, 763-773.

[5] Lin, D.Y., Fleming, T.R. and Wei, L.j. (1994). Confidence bands for survival curves under the proportional hazards model. *Biometrika*, **81**, 73-81.

[6] Xu, Ronghui and 'Quigley, John (2000). Proportional hazards estimate of the conditional survival function. *J. R. Statist. Soc. B*, **62**, 667-680.

RESUME

In this paper we propose a new estimate of the conditional cumulative hazard function given some subset of covariates under a proportional hazards model. This estimate is based on the idea of Xu and O'Quigley(2000) and the asymptotic normality is provided. Further we consider graphical methods for checking proportionality of hazards with respect to two interesting subsets of covariates such as age groups or risk groups, and our result is applied to real data.