

# The Mathematical Series in Synodic Time Scale. Part III

## Time Series of Order 32. Applications and Results.

Sujata Mukherjee  
 B14/168, Kalyani 741 235, WB, India, [sujmuk@hotmail.com](mailto:sujmuk@hotmail.com)

### Summary

The mathematical series, in Synodic time scale, is given as,

$$y_{(L/2-14)}, y_{(L/2-13)}, \dots, y_{(L/2)}, y_{(L/2+1)}, \dots, y_{(L/2+14)} \quad \dots(\text{part I})$$

for one synodic month, where,  $y_{(L/2-14)}$  to  $y_{(L/2+14)}$  are 29 chained variables in increasing and decreasing order of lunar phase, from nomoon to newmoon,  $L/2$  standing for the fullmoon (FM). The applications indicated modification (*STAT' 2000, August 2000, Poland, abs.*).

The mathematical series, for a Synodic month, in modified form, is given as,

$$y_{(L/2-16)}, y_{(L/2-15)}, \dots, y_{(L/2)}, y_{(L/2+1)}, \dots, y_{(L/2+15)} \quad \dots(\text{part II})$$

Where,  $y_{(L/2-16)}$  to  $y_{(L/2+15)}$  are 32 chained variables in increasing and decreasing order of lunar phases (*Stability Problem of Stochastic Model, January-February, 2001, Hungary, abs.*).

In this paper, the mathematical series is expressed in a simple sequential order of 32 phase groups in Synodic time scale, (*part III*), from  $y_{tp1}$  to  $y_{tpS}$  as a Synodic month, given as;

$$y_{tp1}, y_{tp2}, \dots, y_{tp31}, y_{tpS}, y_{tpS+1}, \dots, y_{tp2S}, \dots, y_{tp(n-1)S+31}, y_{tpnS}, \dots \quad \dots(\text{part III})$$

with seasonality  $S = 32$  (*paper accepted, 10<sup>th</sup> IWMS, August 2-3, 2001, The Netherlands*).

Application to published data (*Inter.Conf.P.V.Epid; 1986, Florida, ext. abs. & Indian Soc. for Agricultural Statistics, 2000, 53, 2, 182-186*) in this scale was proved to be *intrinsic* for noctuids (*5<sup>th</sup> BS/IMS, May 2000, Mexico*) the *insight* of which was demonstrated earlier (*4<sup>th</sup> SSC/DST, 1997, WB, India*). The *inferences* on outliers were explained by a *HMM* (*paper accepted for 3<sup>rd</sup> Bayesian nonparametrics summit, July-August 2001, Michigan*) in this scale, influencing generally all species (*paper accepted, ISTAT, CAESAR, June 2001, Rome*) with irregularities probably for not involving transformation based on ecological data (*paper accepted, SPA27, July 2001, Cambridge*). Here this scale is proved to be suitable for *ARIMA models* (*paper accepted for 23<sup>rd</sup> EMS, August 13-18, 2001, Portugal*) for all the published Synodic time series data.

### 1. The Synodic time series; Part I & Part II

The concept of Synodic time scale is based on the Astronomical Ephemeris, where, in the chapter of the physical observation of the Moon, the portion of the lunar disc illuminated by the Sun i.e. the lunar phases, at 00 hours, is given in the last column, against each calendar date of a year. The discrete series  $[y_i]$ , at time  $t_1, t_2, \dots, t_N$ , is given as ;

$$y_{t1}, y_{t2}, \dots, y_{tN} \quad \dots(1.1)$$

The time,  $t$ , is changed to lunar phase  $tp$ , in the discrete series, and is reformulated following the ephemeris, from the beginning of new moon to corresponding no moon, as a Synodic month;

$$y_{tp(.000)}, y_{tp(.002)}, \dots, y_{tp(.039)}, \dots, y_{tp(.098)}, y_{tp(1.00)}, y_{tp(.099)}, \dots$$

$$\dots, y_{tp(.050)}, \dots, y_{tp(.006)}, y_{tp(.005)}, y_{tp(.004)}, y_{tp(.001)}, \dots \quad \dots \text{a Synodic month}$$

which is,  $y_{(L/2-14)}, y_{(L/2-13)}, \dots, y_{(L/2)}, \dots, y_{(L/2+13)}, y_{2(L/2+14)}, y_{2(L/2-14)} \dots$

$$y_{(n-1)(L/2)}, \dots, y_{(n-1)(L/2+14)}, y_{(n)(L/2-14)}, \dots, y_{(n)(L/2)}, \dots, y_{n(L/2+14)} \quad \dots(1.2)$$

where,  $n$  is total # of Synodic months,  $n < N$  &  $n \times 29 = N$ ;  $y_{(L/2-14)}$  to  $y_{(L/2+14)}$  are chained variables in increasing and decreasing order of 29 lunar phases, from no moon to new moon,  $L/2$  standing for the full moon (FM). The mean of the series is  $[\sum y_{sm(tp_i)}]$  divided by  $N$ , for  $1 \leq i \leq 29$  and  $sm$  # of Synodic months,  $N = 29 \times sm$ . The  $k^{\text{th}}$  term is,  $y_{tpk} = y_{r(L/2+v)}$ , where,  $1 \leq r \leq sm$  &  $0 \leq v \leq (\pm)14$ .

The number of phases in a Synodic month varies from 28 phases to 32 phases, because of missing phases and double phases during full and no moon period. Also the waxing and waning period of a Synodic month are unequal. All these points are not covered by the series in (1.2) and modification was proved to be necessary in the numerous previous studies done in this scale.

For modification, these  $[y_{tpi}]$ , are then grouped (Bowden, 1973a, 1973b; Bull. Ent. Res.), inserting missing phases where required. The reformulated series, for a Synodic month, with 32 phase group is,

$$Y_{tp(.000)=newmoon}, Y_{tp(.001-.002)}, Y_{tp(.003-.006)}, \dots, Y_{tp(.098-.099)}, Y_{tp(1.00)=fullmoon}, \\ Y_{tp(.099-.098)}, Y_{tp(.097-.094)}, \dots, Y_{tp(.006-.003)}, Y_{tp(.002.001)=nomoon} \quad \dots(1.3)$$

which is,  $y_{(L/2-16)}, y_{(L/2-15)}, \dots, y_{(L/2)}, \dots, y_{(L/2+15)}, y_{2(L/2-16)}, \dots, y_{2(L/2+15)}, y_{3(L/2-16)}, \dots$

$$y_{(n-1)(L/2-16)}, \dots, y_{(n-1)(L/2+15)}, y_{(n)L/2-16}, \dots, y_{(n)(L/2)}, \dots, y_{n(L/2+15)} \quad \dots(1.4)$$

where, n is total number of Synodic months,  $n < N$  &  $n \times 32 = N$ ;  $y_{(L/2-16)}$  to  $y_{(L/2+15)}$  are chained variables in increasing and decreasing order of 32 lunar phases,  $L/2$  the full moon. The mathematical series (1.4) shows that the two halves of the Synodic month are unequal. This modification makes a Synodic year of 384 phases, gaining 19 data counts than losing 11 in a year of 365 days with a Synodic month of 29.53 days. The mean of the series is  $[\sum y_{SM(tpi)}]$  divided by N, for  $1 \leq i \leq 32$  and # of Synodic months,  $N = 32 \times SM$ . The  $k^{th}$  term is,  $y_{tpk} = y_{r(L/2+v)}$ , where,  $1 \leq r \leq SM$  &  $0 \leq v \leq (\pm)15$ , also  $v = (-)16$ .

### 2. The Synodic time series; Part III

The mathematical series, in 32 phase groups, can be expressed in a simple sequential order of 32 phase groups. For one Synodic month, the series in (1.3) can be written as ;

$$y_{tp(1)}, y_{tp(2)}, \dots, y_{tp(17)}, y_{tp(18)}, \dots, y_{tp(24)}, y_{tp(25)}, \dots, y_{tp(32)} \quad \dots(2.1)$$

and finally,  $y_{tp(1)}, \dots, y_{tp(32)}, y_{tp(32)+1}, y_{tp(32)+2}, \dots, y_{tp(32)+31}, y_{tp(32)+1}, y_{tp(32)+1}, \dots$

$$y_{tp(n-1)(32)}, y_{tp(n-1)(32)+1}, \dots, y_{tp(n-1)(32)+31}, y_{tpn(32)}, \dots \quad \dots(2.2)$$

where,  $y_{tp(1)}$  to  $y_{tp(32)}$  makes a complete Synodic month, from no moon to new moon,  $y_{tp(17)}$  denoting the FM, with seasonality,  $S = 32$ . In this way, a lunar phase is less than 24 hours, in general, but there are exceptions when even double phases occur along with phases having more than 24 hours. The reformulated data series,  $[y_{ti}]$ , thus increases the data counts while time scale is changed from  $t$  to  $tp$ . Adding extra hidden phases increases the number of data points and the out of sample units are used for double phases, if it occurs while forecasting with a Synodic time series model. The general equation of the Synodic time series is;

$$y_{tp1}, \dots, y_{tpS}, y_{tpS+1}, y_{tpS+2}, \dots, y_{tpS+31}, y_{tp2S}, y_{tp2S+1}, \dots$$

$$y_{tp(n-1)S}, y_{tp(n-1)S+1}, \dots, y_{tp(n-1)S+31}, y_{tpnS}, \dots \quad \dots(2.3)$$

The  $k^{th}$  term is,  $y_{tpk} = y_{tp(aS+b)}$ , where, for the first synodic month,  $a=0$  but  $b \geq 1$ ,  $1 \leq a \leq (n-1)$ , &  $1 \leq b \leq 32$ , with  $a = n$  and  $b = 0$  for the last term. The mean of the series is  $\sum y_{n(tpi)}$  divided by N, for  $1 \leq i \leq 32$  and  $N = 32 \times n$  where n is the number of Synodic months.

### 3. Applications.

The patterns of the 7 different Time Series, plotted by FORTRAN & BASIC programs (as in ASMDA, 2001, June 12-15, France), show irregularity in their structure, except for TS 1, 2, 4 & 7. For TS 1 the data counts are quite large and for TS 2, 4 & 7, some other factor/factors may be influencing the pattern to behave less irregularly. All the plotted series, indicate the data to be appropriate, for ARIMA models for Synodic Time Series of order 32.