

How to Relate Robustness Measures with Reliability

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1. Introduction

- Experimental design has become one of the primary tools for achieving quality of manufactured products.
- Because of a division in the disciplines of quality and reliability, this important tool has not been used to extensively to achieve reliability
- In this presentation, we describe how experimental designs can be used to achieve reliability

2. Experiments with failure time data

Example 1 Rolling Ball Bearing Lifetimes from Hellstrand(1989)

Example 2 Light Experiments from Taguchi(1987)

Example 3 Thermostast Experiment from Bullington et al(1993)

Example 4 Drill Bit Experiment

3. Regression Model for failure time data

3.1. Distribution

(1) failure time t takes the nonnegative values

⇒ a standard linear regression modeling of t is not appropriate

- lognormal density function $t \sim LN(\mathbf{m}\mathbf{s}^2)$

Weibull density function

(2) the most obvious choice is the log transformation $Y = \ln(t)$

- $t \sim LN(\mathbf{m}\mathbf{s}^2) \Rightarrow y = \ln t \sim N(\mathbf{m}\mathbf{s}^2)$
- $t \sim Weibull(\mathbf{I}, \mathbf{j})$

⇒ $y = \ln t \sim \text{extrme value distribution}(\mathbf{m} = -\ln \mathbf{I}, \mathbf{s} = \mathbf{j}^{-1})$

3.2. Model

$$Y_i = \ln(t_i) = \underline{x_i^T} \mathbf{b} + \mathbf{se}_i$$

$$= \mathbf{b}_0 + \mathbf{b}_1 x_{i1} + \Lambda + \mathbf{b}_p x_{ip} + \mathbf{se}_i, \quad i = 1, 2, \Lambda, n$$

- the $\{t_i\}$ are the failure times
- $\underline{x_i^T} = (1, x_{i1}, x_{i2}, \Lambda, x_{ip})$ are the vectors of covariate values
- $\underline{\mathbf{b}} = (\mathbf{b}_0, \mathbf{b}_1, \Lambda, \mathbf{b}_p)$ are the vectors of regression parameters
- \mathbf{s} is the scale parameter
- the errors $\{\mathbf{s}_i\}$ are i.i.d.

(Note) $\{\mathbf{s}_i\}$ have standard extreme-value distribution if t_i follows a Weibull distribution

$\{\mathbf{s}_i\}$ have independent $N(0,1)$ distribution if t_i follows a lognormal distribution

4. Likelihood Approach

(1) MLE

- The likelihood function
 - The lognormal regression model

$$L(\mathbf{b}, \mathbf{s}^2) = \prod_{i \in FAIL} (2\pi\mathbf{s}^2)^{-1/2} \exp\left\{-\frac{1}{2}\left[(y_i - x_i^T \mathbf{b})/\mathbf{s}\right]^2\right\} \times \prod_{i \in CEN} \{1 - \Phi[(y_i - x_i^T \mathbf{b})/\mathbf{s}]\}$$

Weibull regression model

$$L(\mathbf{b}, \mathbf{s}^2) = \prod_{i \in FAIL} (1/\mathbf{s}) \exp\left\{-\left[(y_i - x_i^T \mathbf{b})/\mathbf{s}\right]\right\} \times \prod_{i \in CEN} \exp\left\{-\exp\left[-\left[(y_i - x_i^T \mathbf{b})/\mathbf{s}\right]\right]\right\}$$

MLE for $(\hat{\mathbf{b}}, \hat{\mathbf{s}}^2)$ can be obtained by maximizing $L(\mathbf{b}, \mathbf{s}^2)$ with respect to $(\mathbf{b}, \mathbf{s}^2)$

(2) Likelihood ratio tests

- Likelihood ratio tests provide a method for assessing significance of the i th parameter in \mathbf{q}

$\Gamma = -2 \ln \frac{L(\hat{\mathbf{q}}_{(-i)})}{L(\hat{\mathbf{q}})}$, where the i th covariate is dropped from the model and $\mathbf{q}_{(-i)}$ are the corresponding parameters.

- Under $H_0 : \mathbf{q}_i = 0$, Γ is distributed asymptotically as χ^2 (df=1).

5. Analysis of reliability experimental data

5.1. 2^k factorial design

(1) Weibull Regression Model

$$y_i = \ln(t_i) = \mathbf{b}_0 + \mathbf{b}_1 x_{i1} + \mathbf{b}_2 x_{i2} + \mathbf{b}_3 x_{i3} + \mathbf{se}_i, \quad i = 1, 2, \dots, 8$$

MLEs, Likelihood Ratio(LR), and P-values

5.2. 2^{5-2} fractional factorial design(Taguchi)

Light Experiments

(1) Lognormal Regression Model

$$y = \ln(t) = \mathbf{b}_0 + \mathbf{b}_1 A + \mathbf{b}_2 B + \mathbf{b}_3 C + \mathbf{b}_4 D + \mathbf{b}_5 E + \mathbf{b}_{12}(AB) + \mathbf{se},$$

(2) MLEs, Likelihood Ratio(LR), and P-values

5.3. 12-run Plackett-Burman design with ten replicates

Thermostat Experiment

(1) Lognormal Regression Model 1

$$y = \ln(t) = \mathbf{b}_0 + \mathbf{b}_1 A + \mathbf{b}_2 B + \mathbf{b}_3 C + \mathbf{b}_4 D + \mathbf{b}_5 E + \mathbf{b}_6 F + \mathbf{b}_7 G + \mathbf{b}_8 H + \mathbf{b}_9 I + \mathbf{b}_{10} J + \mathbf{b}_{11} K + \mathbf{se}$$

(2) MLEs, Likelihood Ratio(LR), and P-values

(3) Further Analysis

(4) Lognormal Regression Model 2

$$y = \ln(t) = \mathbf{b}_0 + \mathbf{b}_1 A + \mathbf{b}_2 (EH) + \mathbf{b}_3 C + \mathbf{b}_4 D + \mathbf{b}_5 E + \mathbf{b}_6 F + \mathbf{b}_7 G + \mathbf{b}_8 H + \mathbf{b}_9 I + \mathbf{b}_{10} J + \mathbf{b}_{11} K + \mathbf{se}$$

(5) MLEs, Likelihood Ratio(LR), and P-values

5.4. Taguchi Design

Cross Array and failure Time Data(with censoring time of 3000)

(1) Design :

(2) Factors

(3) Use Weibull Regression Model

(4) MLEs, Likelihood Ratio(LR), and P-values

References

William Q. Meeker and Michael Hamada, 'Statistical Tools for the Rapid Development & Evaluation of High-Reliability Products', IEEE Transactions on Reliability, Vol 44, No 2, 1995 June.

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