On Improved Estimators of the Generalized Variance

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1- Main Result

Consider a multivariate normal linear model in its canonical form. Suppose $\underset{\sim}{X} = (X_1, ..., X_k)$ is $p \times k$ matrix with independent columns Xi~Nρ(ξ,Σ) ,and let S be Wp(n,Σ) distributed independently of X. The problem is to estimate $|\Sigma|$ with loss measured by

$$L(\Phi(S,X),\Sigma,x) = b\{\exp(a(\frac{\Phi(S,X)}{|\Sigma|} - 1)) - a(\frac{\Phi(S,X)}{|\Sigma|} - 1) - 1\} \qquad (1.1)$$

where b>0, and a≠0. The constant a,determines the shape of the loss function ,and the constant b serves

$$\frac{1 - e^{-\frac{a}{n+1}}}{a} |S|$$

to scale the loss function .This loss function is known as linex loss function,which was extensively discussed in Zellner(1986). Parsian ,A. and sanjari Farsipour,N.(1993) showed for the case ρ=1 ($|\Sigma|\sigma 2$) the estimator
is not admissible relative to the class of all estimator based on the sufficient statistic (S,X) under linex loss function. As pointed out by Shorrock and Zidek (1974), the problem remains invariant under the full affine group G acting on the space of p×k matrices as

$$X \to AX + B \quad ,x \to Ax + B \quad ,S \to ASA' \ ,\Sigma \to A\Sigma A' \qquad (1.2)$$

where A(p×p) is any nonsingular matrix and B(p×k) is any matrix. Any affine equivariant estimator must be of the form Φ(S)=c|S| ,where c is a constant. The best choice of the constant c that makes the constant risk of Φ(s) a minimum is the unique solution of the following equation

$$E\{c_{n-1}^2 ... c_{n-p+1}^2 (1 - 2ac_n\, c_{n-1}^2 ... c_{n-p+1}^2)^{-\frac{n}{2}-1})\} = e^a (n-1)...(n-p+1) \qquad (1.3)$$

where $c_{n-1}^2 ... c_{n-p+1}^2$ are independent random variable with chi-square distribution. In case p=2we can see the optimal value of cn ,for some value of a in table (1).

Table(1):Value of cn for some a.

a\n	2	3	4	5	6	7	8	9	10
1	.0497	.03271	.0227	.0165	.013				
2	.08304	.06939	.3934	.02741	.02092	.01581			
3	.1022	.08889	.04841	.03226	.0244	.0176	.01489	.01299	
4	.118	.1049	.0556	.0366	.027	.01881	.01595	.01373	
5	.1321	.1192	.06209	.04037	.02941	.02007	.01682	.01446	.0126

we prove the inadmissibility of the best affine equivariant estimator by showing that the estimator

$$\Phi(S,X)=\min\{c_n|S|, c_{n+k}|S+XX'|\} \qquad (1.4)$$

has uniformly smaller risk than cn| S | .

Theorem (1.1): Any estimator of the form $\Phi(S,X)=\Psi(X'S^{-1}X)|S|$ is dominated by the estimator

$$\Phi^*(S,X)=\min\{\Phi(S,X), c_{n+k}|S+XX'|\} \qquad (1.5)$$

provided a>0.

Also, we establish multivariate analogue of Brewster' s sequential (1973) version.
Specifically,denoting the columns of X by X(1) , X(2) ,. ,.X(k) and setting

$$\Psi^{(0)}(S,X)=c_n|S|$$

$$\Psi^{(i)}(S,X)=\min\{\Psi^{(i-1)}(S,X), c_{n+i}|S+X_{(1)}X_{(1)}'+\dots+X_{(i)}X_{(i)}'|\}$$

$$\tilde{\Psi}^{(i)}(S,X)=\min\{\Psi^{(0)}(S,X), c_{n+i}|S+X_{(1)}X_{(1)}'+\dots+X_{(i)}X_{(i)}'|\} \quad i=1,2,\dots,k \qquad (1.6)$$

we have the following result

Theorem (2.1): For every i=1,. .,.k , $\Psi^{(0)}(S,X)$ is dominated by $\tilde{\Psi}^{(i)}(S,X)$, and $\Psi^{(i)}(S,X)$ is better than $\Psi^{(i-1)}(S,X)$. In particular , $\Psi^{(k)}(S,X)$ dominates all other Ψ' s .

Corrollary 1.1: For every i=1,2,. ,k , $R_{y^{(i)}} \le R_{y^{(i-1)}}$

PEREFRENCES

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Shorrock,R.W. and Zidek ,J.V.(1974). An improved estimator of the generalized variance,Technical Report(CRM-446),Universite de Montreal.
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RESUME

Dans ce papies,notre intention est une estimation de paramètre $|\Sigma|$.Naus obtenons la classe des estimatours.