

# Test of Variance Change in a Long-Memory Process

Minjeong, Chu

*Samsung Fire & Marine Insurance, Risk Management Part*  
87, Euljiro 1ga, Choong-Ku, Seoul, Korea  
justasim@dreamwiz.com

Sinsup, Cho

*Seoul National University, Dept. of Statistics*  
San 56-1, Shillimdong, Kwanak-Ku, Seoul, Korea  
sinsup@snu.ac.kr

## 1. Introduction

Many econometric time series have long-memory properties. Since the long-memory can be observed by data obtained from rather a long period, there can be some change in parameter such as variance. We consider testing variance change in a long-memory process.

## 2. Test for the Change in Variance

Consider the variance change model in the following:

$$X_i = \begin{cases} \mu + \sigma u_i, & i = 1, \dots, k^* \\ \mu + \theta u_i, & i = k^* + 1, \dots, n, \end{cases} \quad (1)$$

where the error process  $\{u_i\}$  is a stationary process with zero mean, unit variance and autocovariances satisfying

$$\lim_{n \rightarrow \infty} \sum_{1 \leq i \leq n} \sum_{1 \leq j \leq n} \gamma(i-j)/(n^{2H}L(n)) = 1. \quad (2)$$

$H > 1/2$  implies a long-memory process. Following Gombay et al. (1996), the test of change in variance is based on the functional

$$M_n(\tau) = \frac{1}{\xi_0} \left[ \sum_{1 \leq i \leq [n\tau]} (X_i - \mu)^2 - \tau \sum_{1 \leq i \leq n} (X_i - \mu)^2 \right], \quad \text{where } \xi_0^2 = \text{Var}_{H_0}(X_1 - \mu)^2. \quad (3)$$

For the derivation of the test statistic, Giraitis and Taqqu (1999) showed that the limiting properties of the form

$$\frac{1}{\sqrt{A(n)}} \sum_{1 \leq i \leq [n\tau]} G(X_i), \quad 0 < \tau < 1, \quad (4)$$

only depends on the Hermite rank of  $G$  if the process  $\{X_i\}$  is a stationary Gaussian long-memory process and  $G$  is an arbitrary function, using Taqqu's strong reduction theorem (1975, 1979). Non-Gaussian case is similar to the Gaussian case and the limiting distribution of the test statistic can be derived as follows.

**Theorem 1**

For a strongly dependent process  $\{X_i\}$  satisfying (1) and (2) for  $0 < H < 1$ , assume that  $H_0 : \hat{k}^* > n$  holds. Then there exist non-degenerate processes  $\bar{Z}_2^1(\tau)$  and  $\bar{Z}_2^2(\tau)$  such that

(i) if  $1/2 < H < 3/4$ , then

$$M_n(\tau) \rightarrow \bar{Z}_2^1(\tau) \quad (5)$$

and

(ii) if  $3/4 < H < 1$ , then

$$M_n(\tau) \rightarrow \bar{Z}_2^2(\tau) \quad (6)$$

for all  $0 < \tau < 1$ .

Since the Gaussianity of the limit process and the limit process itself are dependent on the nuisance parameter  $H$ , we cannot obtain standardized critical regions for testing. But if we put

$$\delta = \sigma^2 - \theta^2 \quad (7)$$

and

$$g_n(\tau) = \begin{cases} -(n - k^*)\delta\tau, & 0 \leq \tau \leq k^*/n \\ -k^*\delta(1 - \tau), & k^*/n \leq \tau \leq 1 \end{cases} \quad (8)$$

the consistency of the test statistic can be derived.

**Theorem 2**

For a strongly dependent process  $\{X_i\}$  satisfying (1) and (2) for  $0 < H < 1$ , there exist non-degenerate processes  $\bar{Z}_2^1(\tau)$  and  $\bar{Z}_2^2(\tau)$  such that we have

(i) if  $1/2 < H < 3/4$ , then

$$\frac{1}{\xi_0} \left[ \left\{ \sum_{1 \leq i \leq [n\tau]} (X_i - \mu)^2 - \tau \sum_{1 \leq i \leq n} (X_i - \mu)^2 \right\} - g_n(\tau) \right] \rightarrow \bar{Z}_2^1(\tau) \quad (9)$$

and

(ii) if  $3/4 < H < 1$ , then

$$\frac{1}{\xi_0} \left[ \left\{ \sum_{1 \leq i \leq [n\tau]} (X_i - \mu)^2 - \tau \sum_{1 \leq i \leq n} (X_i - \mu)^2 \right\} - g_n(\tau) \right] \rightarrow \bar{Z}_2^2(\tau) \quad (10)$$

for all  $0 < \tau < 1$  under  $H_A$ .

**REFERENCES**

Giraitis, L. and Taqqu, M. S. (1999) Convergence of normalized quadratic forms. *Journal of Statistical Planning and Inference* **80**, 15-35

Gombay, E., Horváth, L. and Hušková, M. (1996) Estimators and tests for change in variances. *Statistics and Decisions* **14**, 145-159

Taqqu, M. S. (1975) Weak convergence to fractional Brownian motion and to the Rosenblatt process. *Z. Wahrscheinlichkeitstheorie verw. Gebiete* **31**, 287-302

Taqqu, M. S. (1979) Convergence of integrated processes of arbitrary Hermite rank. *Z. Wahrscheinlichkeitstheorie verw. Gebiete* **50**, 53-83