M-estimation of Nonlinear AR Time Series

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1. Introduction

Aase (1983) has dealt with recursive estimation in nonlinear time series of autoregressive type including its asymptotic properties. This contribution modifies the corresponding results for the case of nonlinear time series with outliers using the principle of M-estimation from robust statistics. Strong consistency of the robust recursive estimates is preserved under corresponding assumptions.

A simple model of nonlinear time series of autoregressive type can be written as

$$X_t = \vartheta \, f_t(F_{t-1}^X) + e_t \tag{1}$$

for $t = 1, 2, \ldots$, where $f_t = f_t(F_{t-1}^X)$ is a (nonlinear) real-valued function of the past of the process X_t (i.e. a function of $X_{t-1}, X_{t-2}, \ldots$) such that

$$\mathrm{E}\ f_t^2 \le M \tag{2}$$

for a constant M, ϑ is a real parameter to be estimated and e_t is an error sequence satisfying (conditionally on $X_{t-1}, X_{t-2}, \ldots$ using the symbol F_{t-1}^X again)

$$\mathrm{E}\,(e_t \mid F_{t-1}^X) = 0 \tag{3}$$

$$\mathrm{E}(e_t^2 \mid F_{t-1}^X) = \sigma^2 \tag{4}$$

where $\sigma^2 > 0$ is another unknown parameter of the model. In the model of this type Aase (1983) has derived convergence results for the recursive least squares estimates.

In practice the data that we have in our disposal to estimate (1) can be contaminated by presence of heavy-tailed distributions. In such a case the occurrence of outliers should be respected by using robust statistical procedures.

2. Robust Recursive Estimates

The suggested robust recursive estimates are based on the principle of M-estimation from robust statistics. If one applies a suitable robustifying function ρ to the scaled residuals then after deriving one obtains the normal equation

$$\sum_{k=1}^{t} f_k \psi\left(\frac{X_k - \vartheta f_k}{\sigma}\right) = 0 \qquad (5)$$

with the corresponding psi-function $\psi = \rho'$. Now iterative numerical procedures for M-estimation in time series may be arranged to various recursive forms. One example of possible robust recursive estimates is

$$\hat{\vartheta}_t = \hat{\vartheta}_{t-1} + \frac{f_t k_{t-1}}{1 + f_t^2 k_{t-1}} \sigma_t \psi(r_t / \sigma_t) \qquad (6)$$

$$k_t = k_{t-1} - \frac{f_t^2 k_{t-1}^2}{1 + f_t^2 k_{t-1}} = \frac{k_{t-1}}{1 + f_t^2 k_{t-1}} \qquad (7)$$

where

$$r_t = X_t - \hat{\vartheta}_{t-1} f_t \qquad (8)$$

and σ_t is a suitable robust recursive estimate of the scale parameter σ.

3. Strong Consistency

Strong consistency of the robust recursive estimates of this type can by proved. Similarly as in Cipra and Romera (1991) for linear autoregressive models in the general framework of robust Kalman filtering one can use also in this case properties of supermartingales (see Robbins and Siegmund (1971)).

The robust recursive estimates suggested in this contribution have been compared by means of numerical simulations.

Supported by grant GAÈR 201/00/0770 and MSM 113200008

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RESUME

Un estimateur robust recoursif aux serieux chronologiques nonlineaires du type autoregressif est proposé avec la proprieté de la convergence forte.