Grey Trend Models

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1. Grey Model GM(1,1)$_T$

Grey model GM(1,1)[1] has being widely used in many areas. Based on this grey model, Wang[2] presented a kind of time-varying grey model GM(1,1)$_T$.

Let raw data series $x^{(0)}=(x^{(0)}(1),x^{(0)}(2),\ldots,x^{(0)}(n))$, $x^{(1)}$ be the 1-AGO series generated from $x^{(0)}$, $x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i)$, and $x^{(1)}=(x^{(1)}(1),x^{(1)}(2),\ldots,x^{(1)}(n))$, $x^{(0)}$ and $x^{(1)}$ conform to the definition of the conditions of grey differential equation[1], then the data of $x^{(0)}$ and $x^{(1)}$ satisfy $y_N=B\boldsymbol{a}$, where

$$y_N=\begin{bmatrix}x^{(0)}(2)\\x^{(0)}(3)\\\vdots\\x^{(0)}(n)\end{bmatrix},B=\begin{bmatrix}-z^{(1)}(2)&2-0.5&1\\-z^{(1)}(3)&3-0.5&1\\\vdots&\vdots&\vdots\\-z^{(1)}(n)&n-0.5&1\end{bmatrix},\boldsymbol{a}=\begin{bmatrix}a\\b\\c\end{bmatrix}$$

iff the residual series $\boldsymbol{e}$ submits to the principle of least square sum, that is $J=\boldsymbol{e}^T\boldsymbol{e}=\min$, $\boldsymbol{e}=y_N-B\boldsymbol{a}$, the vector $\boldsymbol{a}=(B^TB)^{-1}B^Ty_N$. Constructing the whitening function with above a,b,c,

$$\frac{\mathrm{d}x^{(1)}}{\mathrm{d}t}+ax^{(1)}=bt+c$$

thenitssolutionis

$$\hat{x}^{(1)}(k+1)=\frac{b}{a}(k+1)-\frac{b-ac}{a^2}+(x^{(0)}(1)-\frac{b}{a}+\frac{b-ac}{a^2})e^{-ak},k=1,2,\cdots,n-1$$
$$\hat{x}^{(1)}(1)=x^{(0)}(1)$$

where $\hat{x}^{(1)}(k+1)$ is the value calculated from model, then its reduced value $\hat{x}^{(0)}(k+1)$ satisfies $\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k)$, that is

$$\hat{x}^{(0)}(k+1)=\frac{b}{a}+(x^{(0)}(1)-\frac{b}{a}+\frac{b-ac}{a^2})(1-e^{a})e^{-ak},k=1,2,\cdots,n-1$$
$$\hat{x}^{(0)}(1)=x^{(0)}(1)$$

2. Grey Gompertz Model

Let the raw data $x^{(0)}=(x^{(0)}(1),x^{(0)}(2),\ldots,x^{(0)}(n))$, and $x^{(0)}(i)>0,i=1,2,\ldots,n$. Transforming $x^{(0)}$ with logarithmic transformation, and let $y^{(0)}(i)=\ln(x^{(0)}(i))$, $i=1,2,\ldots,n$, then we obtain new series

$y^{(0)}=(y^{(0)}(1),y^{(0)}(2),\ldots,y^{(0)}(n))$. According to section 1,construct grey model $GM(1,1)_T$, and the reduced model is

$$\hat{y}^{(0)}(k+1)=\frac{b}{a}+(y^{(0)}(1)-\frac{b}{a}+\frac{b-ac}{a^2})(1-e^{a})e^{-ak},k=1,2,\ldots,n-1$$

$$\hat{y}^{(0)}(1)=y^{(0)}(1)$$

Transforming $\hat{y}^{(0)}$ with exponential transformation, we obtain

$$\hat{x}^{(0)}(k+1)=\exp(\frac{b}{a}+(\ln(x^{(0)}(1))-\frac{b}{a}+\frac{b-ac}{a^2})(1-e^{a})e^{-ak}),k=1,2,\ldots,n-1$$

$$\hat{x}^{(0)}(1)=x^{(0)}(1)$$

The above model is called to be grey Gompertz model.

3. Grey Logistic Model

Let the raw data $x^{(0)}=(x^{(0)}(1),x^{(0)}(2),\ldots,x^{(0)}(n))$,and $x^{(0)}(i)>0,i=1,2,\ldots,n$. Transforming $x^{(0)}$ with reciprocal transformation, and let $y^{(0)}(i)=\frac{1}{x^{(0)}(i)}$, $i=1,2,\ldots,n$, then we obtain new series $y^{(0)}=(y^{(0)}(1),y^{(0)}(2),\ldots,y^{(0)}(n))$. According to section 1,construct grey model $GM(1,1)_T$, and the reduced model is

$$\hat{y}^{(0)}(k+1)=\frac{b}{a}+(y^{(0)}(1)-\frac{b}{a}+\frac{b-ac}{a^2})(1-e^{a})e^{-ak},k=1,2,\ldots,n-1$$

$$\hat{y}^{(0)}(1)=y^{(0)}(1)$$

Transforming $\hat{y}^{(0)}$ with reciprocal transformation, we obtain

$$\hat{x}^{(0)}(k+1)=\frac{1}{\frac{b}{a}+(\frac{1}{x^{(0)}(1)}-\frac{b}{a}+\frac{b-ac}{a^2})(1-e^{a})e^{-ak}},k=1,2,\ldots,n-1$$

$$\hat{x}^{(0)}(1)=x^{(0)}(1)$$

The above model is called to be grey Logistic model.

REFERENCE

[1] Deng Julong. Course of Grey System. Huazhong University of Science and Technology Press, 1990.(in Chinese)

[2] Wang Ziliang. Time-vary grey dynamic model and its characteristics. The Journal of Grey System, 2001,13(3).

RÉSUMÉ

Modèles Tendanciels de Système Gris

Dans ce papier nous démontrons deux nouveaux modéles tendanciels—modèle gris de Gompertz et modèle gris de Logistic, enutilisant la théoré du système gris.